

**JUNIOR MATHS-1A****Max. Marks :75****Section-A****I. Very Short Answer Questions. Answer all Questions.****Each Question carries 'Two' marks****10x2=20M**

1. Find the Domain of function  $\frac{x}{2-3x}$
2.  $f: R \rightarrow R$  and  $f(x) = \frac{1-x^2}{1+x^2}$  then prove that  $f(\tan \theta) = \cos 2\theta$
3. A certain book shop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount that book shop will receive by selling all the books using matrix algebra.
4. If  $\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -x & -7 & 0 \end{bmatrix}$  is a skew symmetric matrix, then find x.
5. If  $4\vec{i} + \frac{2p}{3}\vec{j} + p\vec{k}$  is parallel to the vector  $\vec{i} + 2\vec{j} + 3\vec{k}$ , find p
6. Let  $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$  and  $\vec{b} = 3\vec{i} + \vec{j}$ . Find the unit vector in the direction of  $\vec{a} + \vec{b}$ .
7. Let  $\vec{a} = \vec{i} + \vec{j} + \vec{k}$  and  $\vec{b} = 2\vec{i} + 3\vec{j} + \vec{k}$ . Find the projection vector of  $\vec{a}$  on  $\vec{b}$  and its magnitude.
8. Find the value of  $\sin^2 82^\circ \frac{1}{2} - \sin^2 22^\circ \frac{1}{2}$
9. Find the period of  $f(x) = \sin^4 x + \cos^4 x$  For any  $x \in R$ ,
10. If  $\sinh x = 3$  then prove that  $x = \log(3 + \sqrt{10})$

**Section-B****II. Short Answer Questions. Answer any 'Five' Questions.****Each Question carries 'Four' marks.****5 x4 =20 M**

11. Show that  $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$  Then prove that  $A^2 - 4A - 5I = 0$

11. Prove by vector method that  $\frac{x}{a} + \frac{y}{b} = 1$  is the equation of a straight line in intercept form
12. Find the volume of the tetrahedron whose vertices are (1, 2, 1) (3, 2, 5), (2, -1, 0) and (-1, 0, 1)
13. If  $A + B = \frac{\pi}{4}$ , then prove that  $(1 + \tan A)(1 + \tan B) = 2$ .
14. If  $\tan(\pi \cos x) = \cot(\pi \sin x)$ . Prove that  $\cos\left(x - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$
15. Prove that  $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$
17. Prove that  $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

### Section-C

### III. Long Answer Questions. Answer any 'Five' Questions.

Each Question carries 'Seven' marks.

5 x 7 = 35 M

18. If  $f: A \rightarrow B$  is bijective function then and  $f \circ f^{-1} = I_A$  and  $f^{-1} \circ f = I_B$
19. Prove by Mathematical Induction  $1.3 + 3.5 + 5.7 + \dots$  up to n terms  $= \frac{n(4n^2 + 6n - 1)}{3}$
20. Solve by matrix inverse method
- $$\begin{aligned} 5x - 6y + 4z &= 15 \\ 7x + 4y - 3z &= 19 \\ 2x + y + 6z &= 46 \end{aligned}$$
21. Show that  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$
22. Find the shortest distance between the skew lines  $\vec{r} = (6\vec{i} + 2\vec{j} + 2\vec{k}) + t(\vec{i} - 2\vec{j} + 2\vec{k})$  and  $\vec{r} = (-4\vec{i} - \vec{k}) + s(3\vec{i} - 2\vec{j} - 2\vec{k})$
23. If  $A + B + C = \pi$  prove that  $\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2} = 4 \cos \frac{\pi - A}{4} \cos \frac{\pi - B}{4} \cos \frac{\pi - C}{4}$
24. Show that  $\frac{r_1}{bc} + \frac{r_2}{ac} + \frac{r_3}{ab} = \frac{1}{r} - \frac{1}{2R}$